Question 1  Why do RSA signatures need a hash?

To generate RSA signatures, Alice first creates a standard RSA key pair: \((n, e)\) is the RSA public key and \(d\) is the RSA private key, where \(n\) is the RSA modulus. For standard RSA signatures, we typically set \(e\) to a small prime value such as 3; for this problem, let \(e = 3\).

Suppose we used a simplified scheme for RSA signatures that skips using a hash function and instead uses message \(M\) directly, so the signature \(S\) on a message \(M\) is \(S = M^d \mod n\). In other words, if Alice wants to send a signed message to Bob, she will send \((M, S)\) to Bob where \(S = M^d \mod n\) is computed using her private signing key \(d\).

Q1.1 With this simplified RSA scheme, how can Bob verify whether \(S\) is a valid signature on message \(M\)? In other words, what equation should he check, to confirm whether \(M\) was validly signed by Alice?

Q1.2 Mallory learns that Alice and Bob are using the simplified signature scheme described above and decides to trick Bob into believing that one of Mallory’s messages is from Alice. Explain how Mallory can find an \((M, S)\) pair such that \(S\) will be a valid signature on \(M\).

You should assume that Mallory knows Alice’s public key \(n\), but not Alice’s private key \(d\). The message \(M\) does not have to be chosen in advance and can be gibberish.

Q1.3 Is the attack in Q1.2 possible against the standard RSA signature scheme (the one that includes the cryptographic hash function)? Why or why not?
Question 2  
**RISELab Shenanigans**

Certificate authorities of UC Berkeley are organized in a hierarchy as follows:

```
UC Berkeley
  /      
Campus Buildings  Campus Labs
    /    
Soda Hall  Cory Hall  RISE Lab  ADEPT Lab
```

Alice is a student in RISELab at UC Berkeley and wants to obtain a certificate for her public key. Assume that only RISELab is allowed to issue certificates to Alice.

Q2.1 Which of the following values are included in the certificate issued to Alice? Select all that apply.

- [ ] Alice’s public key
- [ ] Alice’s private key
- [ ] A signature on Alice’s public key, signed by RISELab’s private key
- [ ] A signature on Alice’s private key, signed by RISELab’s private key
- [ ] None of the above

Q2.2 Assume that the only public key you trust is UC Berkeley’s public key. Which certificates do you need to verify in order to be sure that you have Alice’s public key? Select all that apply.

- [ ] Certificate for Alice
- [ ] Certificate for Soda Hall
- [ ] Certificate for RISELab
- [ ] Certificate for Campus Labs
- [ ] None of the above
Q2.3 RISELab issues a certificate to Alice that expires in 1 hour. Which of the following statements are true about using such a short expiration date? Select all that apply.

☐ It mitigates attacks where Alice’s private key is stolen

☐ It mitigates attacks where RISELab’s private key is stolen

☐ It mitigates attacks where Campus Labs’ private key is stolen

☐ It forces Alice to renew the certificate more often

☐ None of the above
**Question 3  Password Storage**

Bob is trying out different methods to securely store users' login passwords for his website.

Mallory is an attacker who can do some amount of *offline* computation before she steals the passwords file, and some amount of *online* computation after stealing the passwords file.

Technical details:

- Each user has a unique username, but several users may have the same password.
- Mallory knows the list of users registered on Bob's site.
- Bob has at most 500 users using his website with passwords between 8–12 letters.
- Mallory's dictionary contains all words that are less than 13 letters. *Clarification during exam: Mallory's dictionary contains all possible user passwords.*
- Mallory can do $N$ online computations and $500N$ offline computations where $N$ is the number of words in the dictionary.
- Slow hash functions take 500 computations per hash while fast hash functions require only 1 computation.1

**Notation:**

- $H_S$ and $H_F$, a slow and fast hash function
- Sign, a secure signing algorithm
- $uname$ and $pwd$, a user’s username and password
- $k$, a signing key known only by Bob

If Bob decides to use signatures in his scheme, assume he will verify them when processing a log-in.

For each part below, indicate all of the things Mallory can do given the password storage scheme. Assume Mallory knows each scheme. **Unless otherwise specified, assume that she can use both offline and online computation**

Q3.1 Each user’s password is stored as $H_F(pwd || 'Bob')$.

- (A) Learn whether two users have the same password with only online computation
- (B) Learn a specific user’s password
- (C) Change a user’s password without detection
- (D) Learn every user’s password
- (E) None of the above

1Keep in mind this is much faster than a real-life slow hash function.
Q3.2 Each user’s password is stored as the tuple \((H_S(pwd || 'Bob'), Sign(k, H_F(pwd)))\).

- [ ] (G) Learn whether two users have the same password with only online computation
- [ ] (J) Learn every user’s password
- [ ] (H) Learn a specific user’s password
- [ ] (K) None of the above
- [ ] (I) Change a user’s password without detection
- [ ] (L) —

Q3.3 Each user’s password is stored as the tuple \((H_F(pwd || uname), Sign(k, uname || H_F(pwd)))\).

- [ ] (A) Learn whether two users have the same password with only online computation
- [ ] (D) Learn every user’s password
- [ ] (B) Learn a specific user’s password
- [ ] (E) None of the above
- [ ] (C) Change a user’s password without detection
- [ ] (F) —

Q3.4 Each user’s password is stored as \((H_S(pwd || uname), Sign(k, H_S(pwd)))\).

[Clarification during exam: The expression was missing a leading parenthesis.]

- [ ] (G) Learn whether two users have the same password with only online computation
- [ ] (J) Learn every user’s password
- [ ] (H) Learn a specific user’s password
- [ ] (K) None of the above
- [ ] (I) Change a user’s password without detection
- [ ] (L) —